Measuring the apparent size of the moon with a digital camera.

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Abstract
The Moon appears to be much larger closer to the horizon than when higher in the sky. This is called the 'Moon Illusion' since the observed size of the Moon is not actually larger when the Moon is just above the horizon. This article describes a technique for verifying that the observed size of the Moon in not larger on the horizon. The technique can be easily performed in a high school teaching environment. Moreover, the technique demonstrates the surprising fact that the observed size of the Moon is actually smaller on the horizon due to atmospheric refraction. For the purposes of this paper, several images of the moon were taken with the Moon close to the horizon and close to the zenith. Images were processed using a free program called ImageJ. The Moon was found to be $5.73 \pm 0.04\%$ smaller in area on the horizon then at the zenith.

Introduction

It is commonly known that the Moon appears larger when it is close to the horizon; however the physical and psychological reasons for the phenomena are not well understood by the general public. Common misconceptions include that the Moon is physically closer to the Earth when it is near the horizon, that the apparent change in size is due to the elevation of the eyes, or atmospheric refraction.

The observed size of the moon is in fact smaller when close to the horizon, the exact opposite of what our senses suggest. The reason is that atmospheric refraction distorts the moon in the vertical direction making it appear oblate rather than circular (Floor 1982). The atmosphere has a different refractive index from space and acts like a lens. More specifically, the atmosphere consists of multiple layers with slightly different refractive indices, each of which refracts the incoming light as depicted in figure 1.

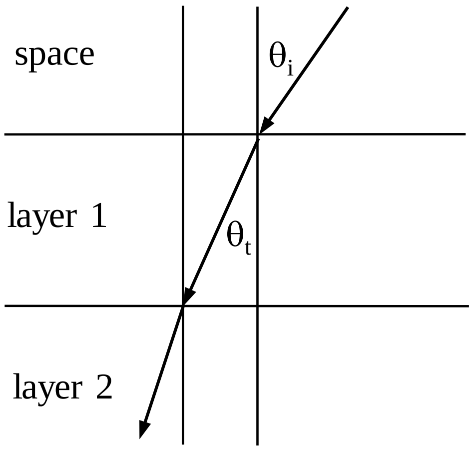

Figure 1. The atmosphere consists of multiple layers. Light entering the atmosphere is refracted at the interface between layers according to Snell's Law.

In this paper we outline a simple experiment which can be used to demonstrate the influence of atmospheric refraction on the Moon. This experiment is low cost and can be easily performed in a high school classroom environment by the students themselves. Students will learn the value of scientific method in testing their preconceived ideas.

Method

Two sets of images were collected, one set using a Casio Exilim ZX2000 14-megapixel camera mounted on a tripod and the other set with a hand-held Canon Powershot SX220 HS 12-megapixel camera. Tracking of the Casio camera was not required as the longest exposure was only 1/8 s. With the Casio camera, 30 pictures of the moon where taken shortly after Moonrise, and another thirty were taken at an elevation of approximately 60°. The photographs were taken with a small exposure time determined automatically by the camera's internal software. The *f*-number was the same for the horizon and zenith Moons to ensure that the scale of each photo was identical. The camera's self-timer function was used to ensure that the camera did not move whilst the shutter was open. The Canon was used to take a single image of the Moon close to the horizon and another when the Moon was at about 60° above the horizon.

Data Analysis

Horizon and zenith images of the Moon were loaded into ImageJ (www.rsbweb.nih.gov/ij/) and an elliptical region of interest was manually fitted to the edges of the Moon, as shown in figure 2. The Measure tool in ImageJ was used to calculate the areas within the regions. Thirty or so measurements for each elevation were used to quantify the error. The ellipse fitted to the image of the Moon close to the zenith was transferred onto the horizon image, as seen in figure 2. This clearly demonstrates that the shape of the Moon close to the horizon is different than at the zenith and is smaller.

Figure 2. A digital image of the moon close to the zenith (left) was enclosed by a circle using ImageJ, (exposure: 1/8 second at f/6.5). A circle the same size was placed over an image of the Moon close to the horizon (exposure 1/125 second at f/6.5). As can be seen, close the horizon the moon is elliptical in shape, and therefore the lower section of the Moon is 'missing'.

The 'Measure' function in ImageJ was used to measure the mean and standard deviation of the area of ellipses manually fitted to the horizon and zenith images,

$$A_{Horizon} = 16572 \pm 62\, pixels^2 \text{ , (1)}$$

$$A_{Zenith} = 17579 \pm 44\, pixels^2 \text{ . (2)}$$

The apparent percentage change in the area (ΔA) was measured to be:

$$\Delta A = 5.73 \ \pm 0.04\ \% \qquad (3)$$

The width and height of the horizon Moon was 151 and 139.5 pixels respectively and the dimensions of the zenith Moon 152.5 and 148 pixels. Figure 3 shows a bar graph of the areas of the horizon and zenith Moons. The error bars are the standard deviations. Note that the difference is highly significant since the error bars are so far apart. The difference between the means is over 16 times greater than the largest standard deviation.

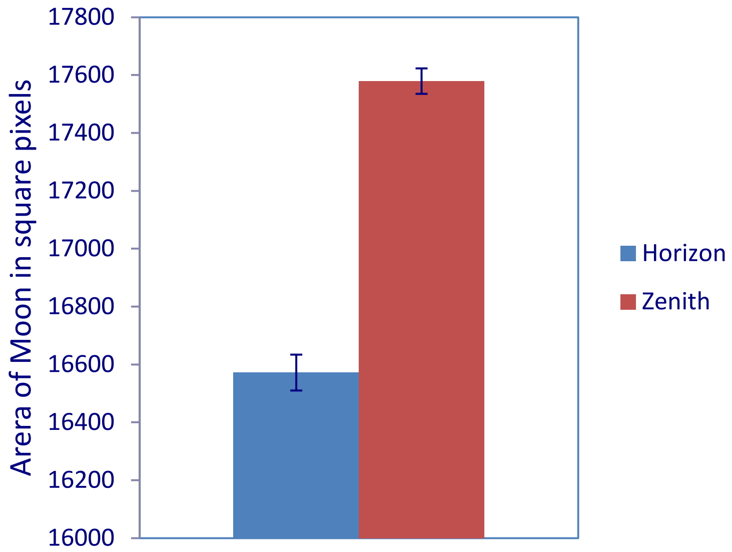

Figure 3. A bar graph of the area of the horizon and zenith Moons. The error bars are the standard deviations of the area measurements.

A students *t*-test revealed that the difference between the means is highly significant with a *p* value close to zero, which means that the probability of the difference in the area between the horizon and zenith Moons being due to chance is astronomically small.

Figure 4. The two images of the Moon taken with the Canon Power Shot SX220 HS. The photo of the 'horizon' Moon on the left was taken with an exposure of 1/15 s at f/5.9 and the photo of the 'zenith' Moon (actually at an elevation of about 60°) was taken with an exposure of 1/125 s at f/5.9. The Moon is an orange colour due to scattering of blue light in the atmosphere. The two photographs of the Moon have been drawn at the same scale. Note that although the width of the two Moons is the same, the heights are different.

Figure 4 shows the two images of the Moon taken with the Canon Power Shot SX220 HS. The photo of the 'horizon' Moon on the left was taken with an exposure of 1/15 s at f/5.9 and the photo of the 'zenith' Moon (actually at an elevation of about 60°) was taken with an exposure of 1/125 s at f/5.9. The Moon is an orange colour due to scattering of blue light in the atmosphere. The two photographs of the Moon have been drawn at the same scale. Note that although the width of the two Moons is the same, the heights are different.

The area of the horizon Moon was 127 996 square pixels and the area of the zenith Moon 135 880, a difference of 5.8%, nearly identical to the Casio result. The width and height of the horizon and zenith images were 416.8, 382.1 and 417, 417 respectively. The width and height of the Moon close to the horizon can be measured with reference to the horizon. However, close to the zenith the camera will most probably not be aligned with the horizon. In this case, several measurements of the diameter can be made to verify the Moon is circular. The original Casio and Canon images shown in this paper can be downloaded from supplementary material (on this ePrints site).

Discussion

Our results show that a digital camera can be used to measure the physical change in size of the moon between the horizon and zenith with a fair degree of accuracy. Although the moon appears several times larger when close to the horizon, the observed size is actually physically smaller by around 5.73 ± 0.04%. This compares well with the results of earlier investigators. (Restle 1970, Taylor and Boring 1942, Georg Von 1949) Theory suggests that the moon could be as much as 17% smaller, but the exact amount will vary depending upon the weather conditions on the day of the experiment.

The experimental procedure described in this article gives students an insight into the very important topics of experimental technique and experimental error. For example, care needs to be taken to ensure that the edges of the Moon can be clearly seen on photographs. If the exposure time is too long, the edges of the moon can blur creating an appreciable error. Some cameras are sensitive enough so that sharp images of the Moon can be obtained with the camera held by hand (as shown in figure 4). However, in some cases a tripod is necessary. If a tripod is used, cameras that can be controlled with a laptop are ideal, as are those that can be remote controlled. If neither option is available the camera self-timer function can be used. The full Moon is bright enough so that short exposures can be used so that there is very little blurring due to the rotation of the Earth. The Moon moves at approximately 0.5° every two minutes, i.e. about one Moon diameter every two minutes. The diameter of the Moon in figure 2 is about 150 pixels, therefore the Moon moves 150 pixels in 120 s, or 1.25 pixels s^{-1}. The movement of the Moon in 1/8 s is just over 1/8 pixel.

CCD sensors tend to be flat rather than spherically curved like the sky, and therefore there will be increasing distortion further from the centre of the CCD. However, if the images of the Moon are in about the same place on the CCD sensor the error in measuring the change in area will be small.

The experiment would also be useful in teaching school students the value of the scientific method in testing preconceived ideas, especially the Moon Illusion that defies common sense. Students should be encouraged to offer their own explanations of the phenomenon both before and after doing the experiment. Another very important point is that students actually get to make their own measurements rather than rely on a book or the internet.

This experiment could be extended to include several altitudes at known increments so that the change in physical size as the moon rises in the sky can be quantified. Students could be asked to conduct a survey amongst themselves or others to compare the physical and perceived change in size of the Moon at different elevations. If multiple measurements are available a statistical test (e.g. *t*-test) could be performed to assess the significance of the difference between the horizon and zenith Moons. Students can be asked test the null hypothesis that the apparent size of horizon and zenith Moons are the same. Some students may expect to disprove the hypothesis by showing that the apparent size of the Moon is greater at the horizon than at the zenith, but then discover the reverse!

Another important aspect of the experiment described in this article is that it is low cost. Over the last few years digital cameras have become available with sufficient resolution (i.e. more megapixels) to accurately measure the area of the Moon without using a telescope. Free image analysis programs such as ImageJ, GIMP and Paint Shop are also readily available.

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